

# Chapter 24

## Solids and polyhedra

### Contents:

- A** Types of solids
- B** Freehand drawings of solids
- C** Isometric projections
- D** Constructing block solids
- E** Nets of solids



### OPENING PROBLEM



Alongside is a sketch of a **solid cone**. It has a circular base and a **curved surface** up to a point called its **apex**.

**Things to think about:**

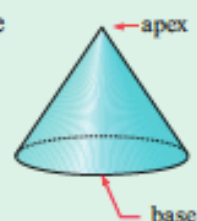
- How can we make a cone from a piece of timber?
- How can we make a model of a cone from a sheet of paper?
- If we make a model from paper, how do we make the cone be flat like



or pointy like

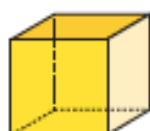


?



A **solid** is a body which occupies space.

These diagrams show some special solids. You should learn their names and be able to draw neat freehand sketches of them.



cube



square-based pyramid



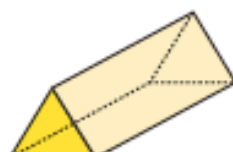
cone



cylinder



sphere



triangular prism

The dashed lines show edges which are hidden at the back of the solid. The dashed lines remind us these edges are there, even if we cannot normally see them.



The boundaries of a solid are called **surfaces**. These surfaces may be flat surfaces, curved surfaces, or a mixture of both.

Which of the above solids have only flat surfaces, only curved surfaces or a combination of both types?

Click on the icon to obtain models of the solids above. Rotate them to help you appreciate their 3-dimensional nature.

DEMO



### DID YOU KNOW?



Did you know that bronze statues are generally hollow inside? This is mainly because bronze is very expensive.



## A

## TYPES OF SOLIDS

## POLYHEDRA

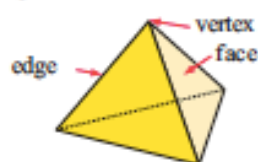
A **polyhedron** is a solid which contains all flat surfaces.  
The plural of polyhedron is **polyhedra**.

Cubes and pyramids are examples of polyhedra. Spheres and cylinders are not.

Each flat surface of a polyhedron is called a **face** and has the shape of a polygon.

Each corner point of a polyhedron is called a **vertex**.

Each intersection of two faces is called an **edge**.



The solid opposite is a triangular-based pyramid, often called a **tetrahedron**.

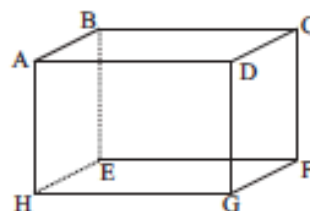
Labelling a figure helps describe its features.

For example:

A, B, C, D, E, F, G and H are all vertices of this polyhedron.

ABCD is one face. There are five other faces.

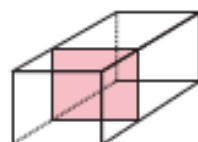
[AB] is one edge. There are eleven other edges.



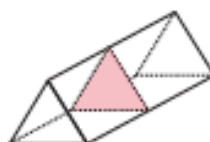
## PRISMS

A **prism** is a polyhedron with a uniform cross-section.

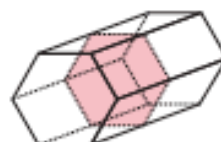
Examples of prisms:



rectangular prism

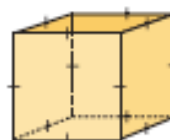


triangular prism



hexagonal prism

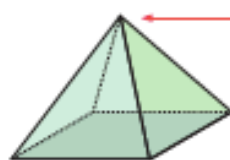
A **cube** is a rectangular prism with 6 square faces.  
All of its edges are the same length.



## PYRAMIDS

A **pyramid** is a solid with a polygon for a base, and triangular faces which come from the base to meet at a point called the **apex**.

Examples of pyramids:



square-based pyramid



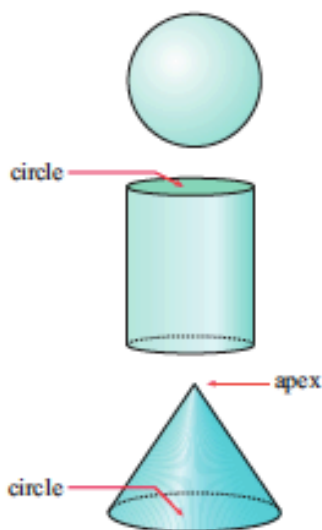
triangular-based pyramid

## SOLIDS WITH CURVED SURFACES

A **sphere** is a perfectly round ball.

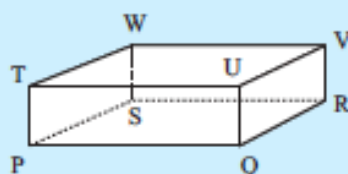
A **cylinder** is a solid with a uniform circular cross-section.

A **cone** has a circular base and a curved surface which rises up to a point called its **apex**.



### Example 1

### Self Tutor

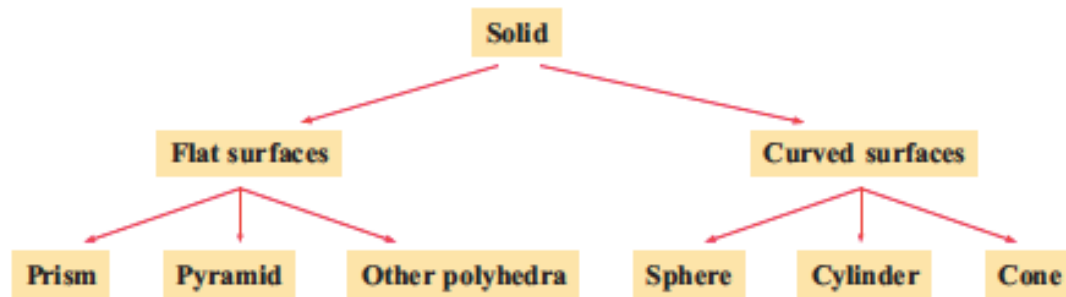


- Classify the given solid.
- Name all of its vertices.
- List all of its edges.
- What face is closest to us?

- A rectangular prism {all faces are rectangles}
- P, Q, R, S, T, U, V and W are its vertices.
- Its edges are: [PQ], [QR], [RS] and [SP] {on the base}  
[PT], [QU], [RV] and [SW] {verticals}  
[TU], [UV], [VW] and [TW] {on the top}.
- The face PQUT is closest to us.

## CLASSIFYING SOLIDS

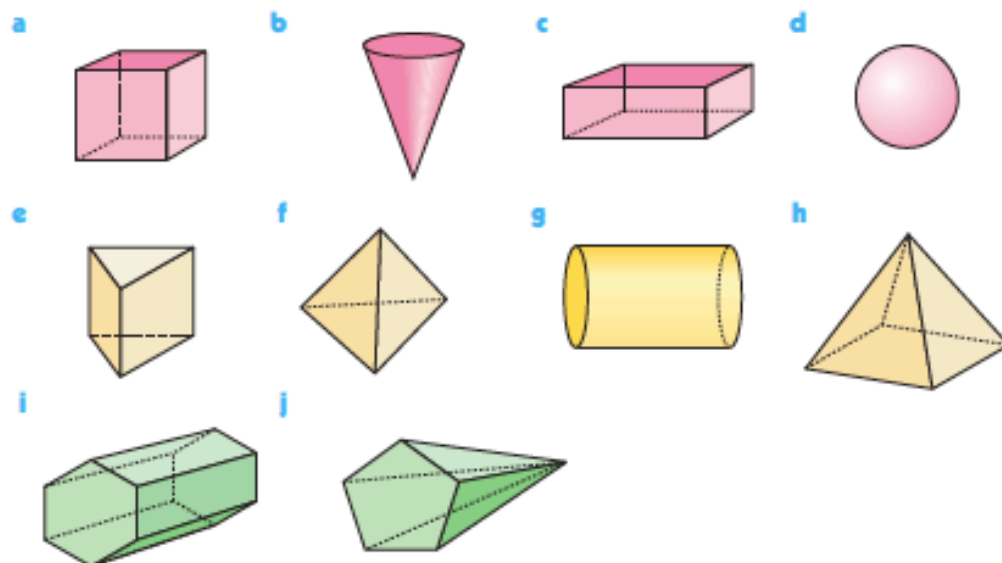
The following flowchart gives us a way of classifying solids:



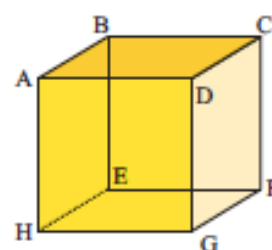
# EXERCISE 24A

- 1 Draw a neat diagram to represent a:
  - a cube
  - b cone
  - c cylinder
  - d sphere
  - e rectangular prism
  - f triangular-based pyramid
- 2 Name the solid which best resembles:
  - a a basketball
  - b the top part of a funnel
  - c a tennis ball container
  - d a six-faced die
  - e a cereal box
  - f a broom handle

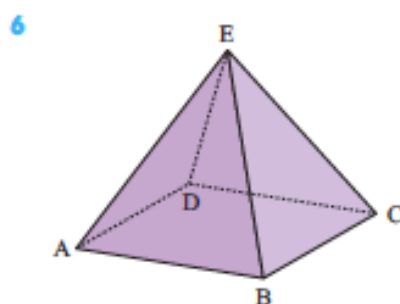
- 3 Classify these solids:



- 4
  - a Name all the vertices of this cube.
  - b Name all of its faces.
  - c Name all of its edges.



- 5 What shapes are the side faces of:
  - a a prism
  - b a pyramid?



For the given pyramid, name and count the:

- a faces
- b vertices
- c edges.

**B****FREEHAND DRAWINGS OF SOLIDS**

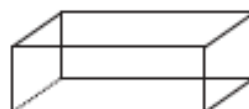
Making freehand sketches of special solids is not easy. Following are step by step instructions on how to do them accurately.

**RECTANGULAR PRISM***Step 1:*

Draw a rectangle for the **front face**.

*Step 2:*

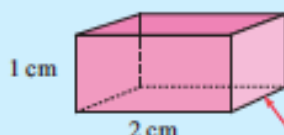
From each of the vertices draw lines back to create the edges. Their lengths are drawn slightly shorter than their real length to give perspective.

*Step 3:*

Complete the drawing by joining the appropriate vertices. Use dotted lines for the hidden edges.

**Example 2****Self Tutor**

Draw a rectangular prism 2 cm long by 1 cm wide by 1 cm high.



this line is drawn shorter than 1 cm

We call this a  $2 \times 1 \times 1$  rectangular prism. The first measurement is length, the second is width and the third is height.

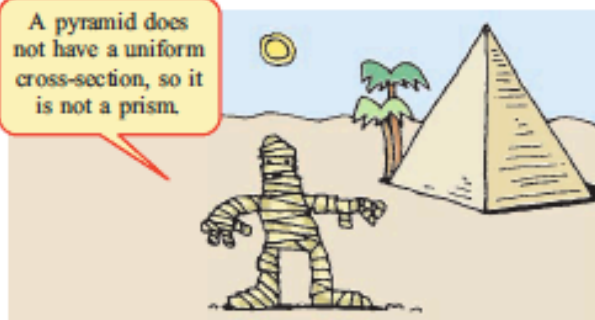
**PYRAMIDS**

In the picture of the pyramid alongside, only five edges, four vertices and two faces can be seen.

In fact, this pyramid has a square base and four triangular faces.

To draw a pyramid we use the following steps:

A pyramid does not have a uniform cross-section, so it is not a prism.



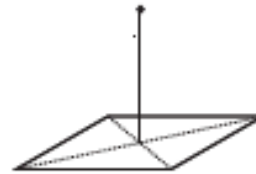


Step 1:



Draw a parallelogram to represent the base.

Step 2:



To find the centre of the parallelogram, draw its diagonals and find their point of intersection. Draw a point above the centre to represent the **apex** of the pyramid.

Step 3:



Join each vertex of the base to the apex to complete the pyramid.

Step 4:



Looking at the picture of the pyramid above, not all edges can be seen at the one time. We show hidden edges as dotted lines.

## CYLINDERS

You are probably familiar with cylinders such as tin cans. You would be aware that their top and bottom is a circle, but when we look at it on an angle it will *appear* as an **ellipse** or oval.

To draw a cylinder we use the following steps.

Step 1:



Draw an ellipse to represent the base.

Step 2:



Draw the sides of the cylinder from the "ends" of the ellipse.

Step 3:



Complete the cylinder by drawing another ellipse on the top.

Step 4:



We use a dashed curve to show the part of the base that is hidden.

## CONES

Just like a cylinder, we represent the circular base of a cone using an ellipse.

To draw a cone we use the following steps:

Step 1:



Draw an ellipse to represent the base.

Step 2:



Mark a point directly above the centre of the ellipse. This will be the apex of the cone.

Step 3:



Join the "ends" of the ellipse to the apex to complete the cone.

Step 4



We use a dashed curve to show the part of the base that is hidden.

## ACTIVITY 1

## VIEWING SOLIDS



In this activity we revisit the demonstration of 3-dimensional solids.

**What to do:**



- 1 Click on the icon to run the software.
- 2 Rotate each of the objects so you can view them from:
  - a directly above      b directly below      c directly alongside.
 In each case, sketch your results.

## EXERCISE 24B

- 1 Draw a rectangular prism that is:
  - a  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$       b  $1\text{ cm} \times 2\text{ cm} \times 1\text{ cm}$       c  $2\text{ cm} \times 2\text{ cm} \times 1\text{ cm}$
- 2 Draw freehand sketches of:
  - a a square-based pyramid      b a tetrahedron
  - c a hexagonal prism      d a triangular-based prism
  - e a hexagonal-based pyramid.
- 3 Draw freehand sketches of:
  - a a cylinder which is 3 cm high and has a base that is 2 cm wide
  - b a cone which is 4 cm high and has a base that is 3 cm wide.
- 4 Sketch a sphere. Use shading to show how it curves.

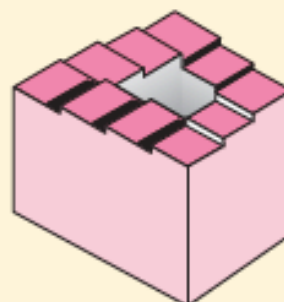
## VISUAL ILLUSIONS



Trigon



Eat from this fork!



Ever-ascending steps

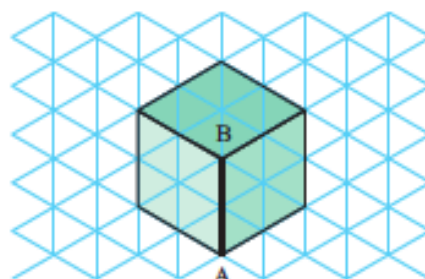


## C

## ISOMETRIC PROJECTIONS

When drawing a rectangular object, we can also use an **isometric projection**. This uses special graph paper made up of equilateral triangles.

The diagram alongside shows the isometric projection of a cube. The edge [AB] appears closest to us, and this is often the **starting edge** of the figure, or the first edge drawn.

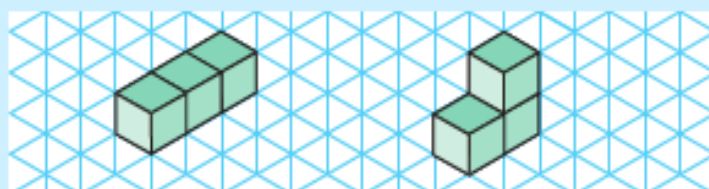


## Example 3

On isometric graph paper, draw the only two different shapes which can be made from three cubes of the same size and which have at least one face in full contact with one of the other cubes.

## Self Tutor

PRINTABLE  
ISOMETRIC PAPER



## EXERCISE 24C

- 1 Redraw the following figures on isometric graph paper:

a



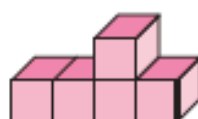
b



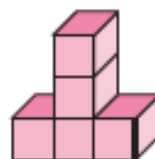
c



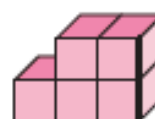
d



e

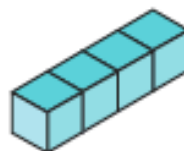


f



- 2 On isometric paper draw all possible different shapes which can be made from four cubes of the same size and which have at least one face in full contact with one of the other cubes.

Note that



and



are the same.

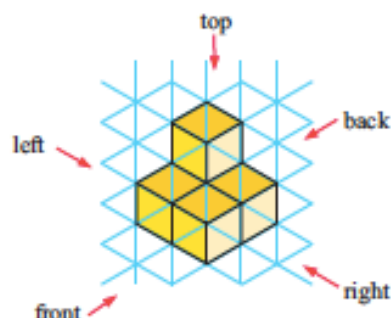
## D

## CONSTRUCTING BLOCK SOLIDS

When an architect draws plans of a building, separate drawings are made from several viewing directions.

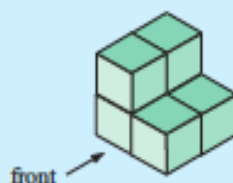
Given a drawing on isometric graph paper, there are 5 directions we consider:

The top view is also called the **plan**. We use numbers on the plan to indicate the height of each pile.



## Example 4

Draw top, front, back, left and right views of:



Self Tutor



The views are:

2	1
2	1

top



front



back



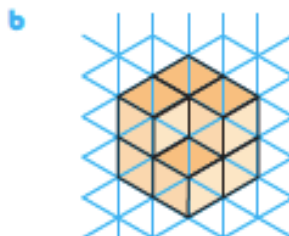
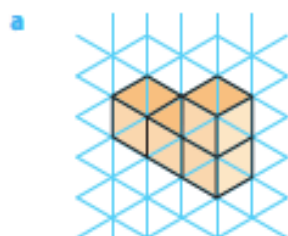
left



right

## EXERCISE 24D

- 1 Draw top, front, back, left and right views of:



## Example 5

The given diagrams show different views of the same shape:

2	1
1	

top



front



back



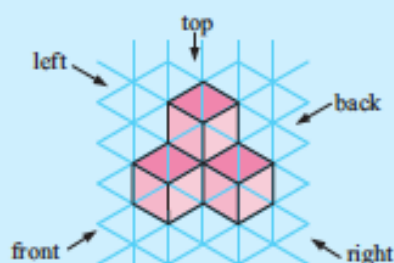
left



right

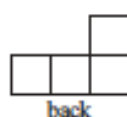
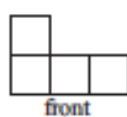
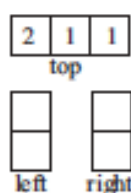
Draw the object on isometric paper.

Self Tutor

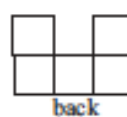
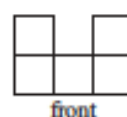
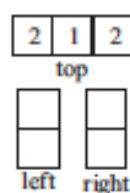


- 2 Draw the 3-dimensional object whose views are:

a



b



- 3 Draw four different objects made from five cubes whose view from the top is 

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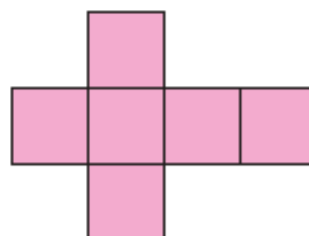
. They must be free standing and not glued together.

## E

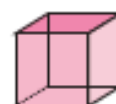
## NETS OF SOLIDS

A **net** is a two-dimensional shape which may be folded to form a solid.

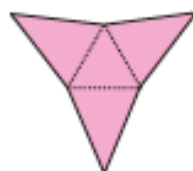
For example, a **cube** is formed when the “net” shown is cut out and folded along the dotted lines.



becomes



A **triangular-based pyramid** is formed when this “net” is cut out and folded along the dotted lines.



becomes



A **square-based pyramid** is formed when this “net” is cut out and folded along the dotted lines.



becomes

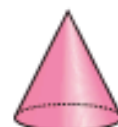


All the examples above are polyhedra and have **flat surfaces**. However, some solids with **curved surfaces** can also be made from nets.

For example, the curved surface of a **cone** is formed from a “net” which is part of a circle. A separate circle can then be cut out to be the base of the cone.



becomes



## ACTIVITY 2

## MAKING MODELS OF SOLIDS



Click on the solid for which you want a printable net. If possible print it on light card rather than ordinary paper.

**What to do:**

- 1 Construct the solids from the nets provided.
- 2 Make a mobile from the solids to hang in your classroom.

PRINTABLE TEMPLATE



## EXERCISE 24E

- 1 Match the net given in the first column with the correct solid and the correct name:

Net	Solid	Name
a	A	(1) Pentagonal-based pyramid
b	B	(2) Triangular prism
c	C	(3) Square-based pyramid
d	D	(4) Cylinder

- 2 Click on the icon to obtain nets for the solids in 1. They have extra tabs to help you glue the solid together.

PRINTABLE NETS



- 3 Is a possible net for a triangular-based pyramid?

## ACTIVITY 3

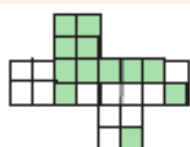
## WHICH CUBE IS IT?

**What to do:**

For each of the cases following:

- 1 Carefully study the nets and the sets of cubes given.
- 2 Determine which cube can be made from the net and write down your answer.
- 3 Construct an actual net showing the **exact** same patterns on the faces. Make the cube and hence check your answer to 2.

Case 1:



P



Q



R



S



T



Case 2:



P



Q



R



S



T



## KEY WORDS USED IN THIS CHAPTER

- apex
- cylinder
- isometric projection
- prism
- sphere
- triangular prism
- cone
- edge
- net
- pyramid
- square-based pyramid
- vertex
- cube
- face
- polyhedron
- solid
- tetrahedron

LINKS  
click here

## PLATONIC SOLIDS

Areas of interaction:  
Human ingenuity, Approaches to learning

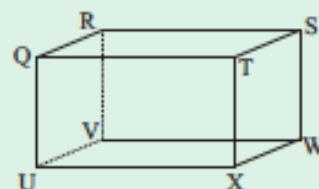
## REVIEW SET 24A

1 Draw the following solids:

- a** a cube                      **b** a cone.

2 For the rectangular prism shown, name all of the:

- a** vertices    **b** faces    **c** edges.

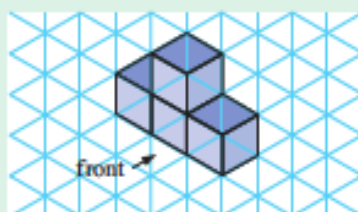


3 Draw a freehand sketch of a cylinder which is 45 mm high and has a base 12 mm wide.

4 Draw the following object as an isometric projection:



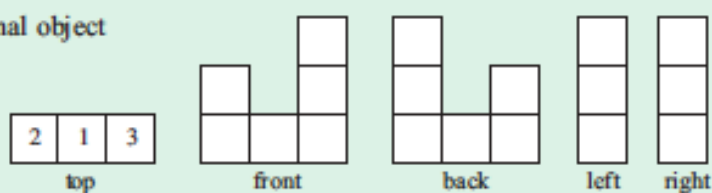
5



For the solid shown, draw views from the:

- a** top                      **b** left  
**c** right                  **d** front  
**e** back.

- 6 Draw the 3-dimensional object whose views are:



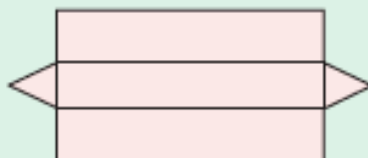
- 7 Draw a net for:

a a rectangular prism

b a cube

c a triangular-based pyramid

- 8 Draw and name the solid corresponding to the net:



### REVIEW SET 24B

- 1 Draw the following solids:

a a triangular prism

b a cylinder

- 2 For the given figure, name:

a all vertices

b all edges

c all faces.



- 3 Draw a freehand sketch of:

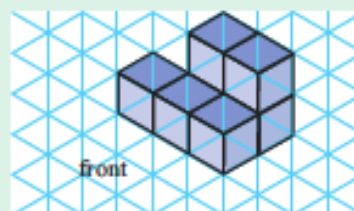
a a cube

b a  $2\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$  rectangular prism.

- 4 Draw an isometric diagram of the solid with views:



- 5 Draw top, front, back, left and right views of:

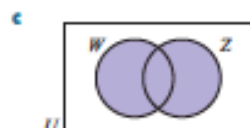
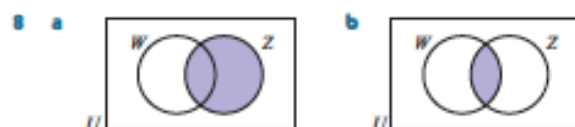


- 6 On isometric graph paper, draw 5 of the possible arrangements of 4 identical blocks where every block is in full contact with at least one full face of another block.
- 7 Draw a net for:
- a a cone
- b a square-based pyramid.
- 8 Draw and name the solid which corresponds to the following net:



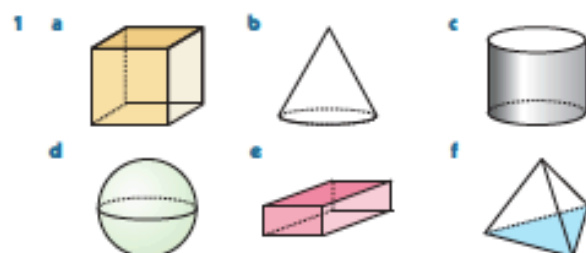


- 7 a The elements in both set  $M$  and set  $N$ .  
 b The elements in set  $N$ , but not set  $M$ .  
 c The elements in neither set  $M$  nor set  $N$ .



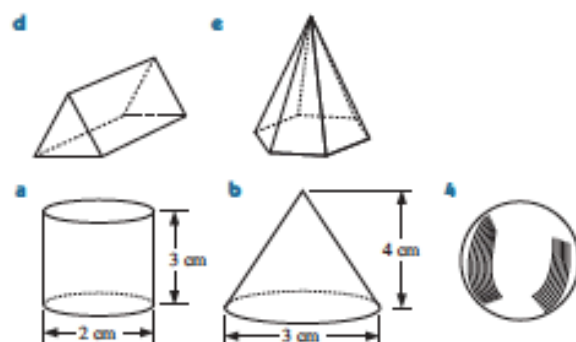
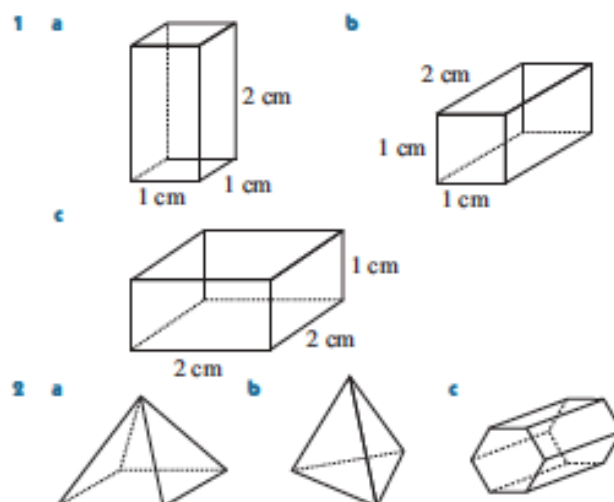
- 9 a  $x = 8$   
 b i 19 ii 24 iii 8 iv 11 v 35 vi 27

## EXERCISE 24A

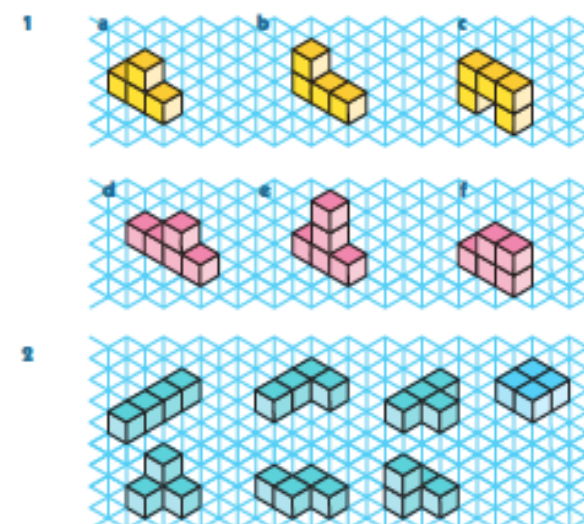


- 2 a sphere b cone c cylinder d cube  
 e rectangular prism f cylinder  
 3 a a cube b a cone c a rectangular prism  
 d a sphere e a triangular prism  
 f a tetrahedron or triangular-based pyramid  
 g a cylinder h a square-based pyramid  
 i a hexagonal prism j a pentagonal-based pyramid  
 4 a A, B, C, D, E, F, G, H  
 b ABCD, BCDE, CDGF, ADGH, ABEH, EFGH  
 c [AB], [DC], [GF], [HE], [BC], [AD], [EF], [HG], [AH], [BE], [CF], [DG]  
 5 a rectangles b triangles c a 5 b 5 c 8

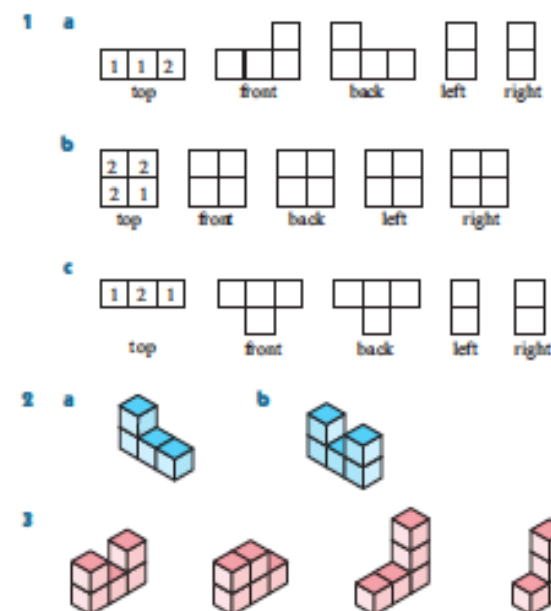
## EXERCISE 24B



## EXERCISE 24C



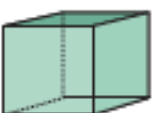

## EXERCISE 24D








## EXERCISE 24E

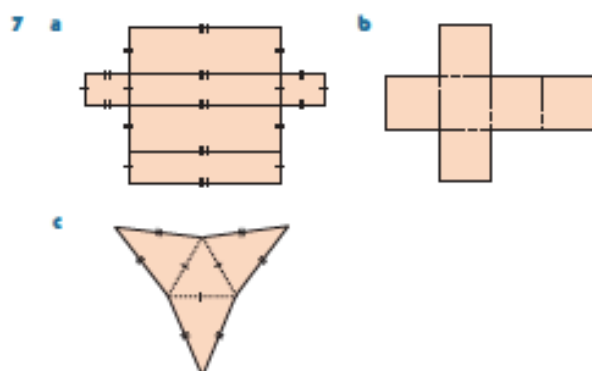
- 1 a C, (2) b A, (3) c B, (4) d D, (1)  
 2 Yes! Make it!

## REVIEW SET 24A

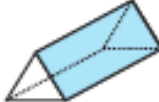

- 1 a  b 
- 2 a U, V, W, X, Q, R, S, T  
 b UVWX, QRST, QTXU, RSWV, QRVU, TSWX  
 c [QR], [RS], [ST], [TQ], [QU], [RV], [SW], [TX], [UV], [VW], [WX], [XU]



- 5 a  b  c   
 d  e 



## REVIEW SET 24B

- 1 a  b 
- 2 a A, B, C, D, E, F  
 b [AB], [AC], [BC], [AE], [BF], [CD], [DF], [DE], [EF]  
 c ABC, EFD, ABFE, ACDE, BCDF

